Common Errors in Linear Regression

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Name:	Role in Meeting:
Ruta Brazauskas, PhD	Planning Committee
Haley Montsma, BBA	Planning Committee
Kwang Woo Ahn, PhD	Speaker

Learning Objectives

- Determine the main components of linear regression
- Utilize simple graphs to check linear regression assumptions
- Identify other common errors

Evaluation Forms

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Thank you.

Linear regression

- Linear regression analysis is a statistical method for investigating linear relationships between response variable and explanatory variables. For example,
  - $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon$.
  - Y is a response variable, X_i is an explanatory variable, ϵ is an error term, and β_i is parameters we need to estimate.
- Explanatory variables are also called independent variables, covariates, regressors, and predictors.
- A response variable is also called a dependent variable.
- In linear regression, the main focus is estimating β_i's and seeing whether they are significant.

Interpretation

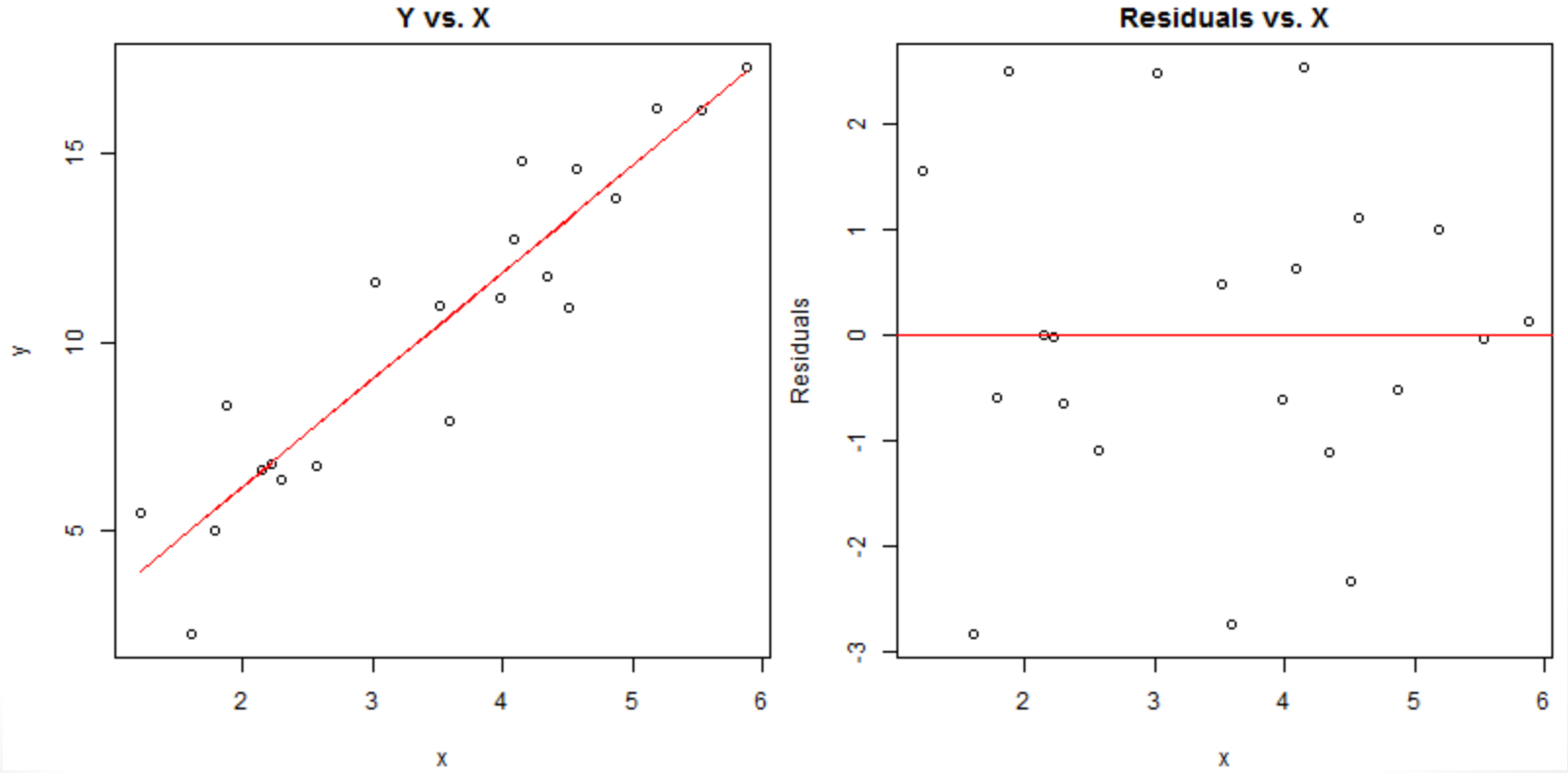
- Assume that we investigate the relationship between rat's length (response variable) and weight/temperature (predictors).
- Consider the following model:
  - Length (mm) = 5 + 2*weight (g) + 0.3*temperature (F).
  - 1 gram increase of weight adds 2mm of length when temperature is held fixed.
  - 1 degree increase of temperature adds 0.3 mm of length when weight is held fixed.

Assumption #1 - Linearity

- Linear regression assumes that the relationship between Y and X_i's is linear.
- Residuals = Observed – Fitted.
- To investigate the linearity assumption, check
  - Plot of Y vs. X_i
  - Scatter plot of residual vs. X_i for all i.
  - Scatter plot of residual vs. the fitted values

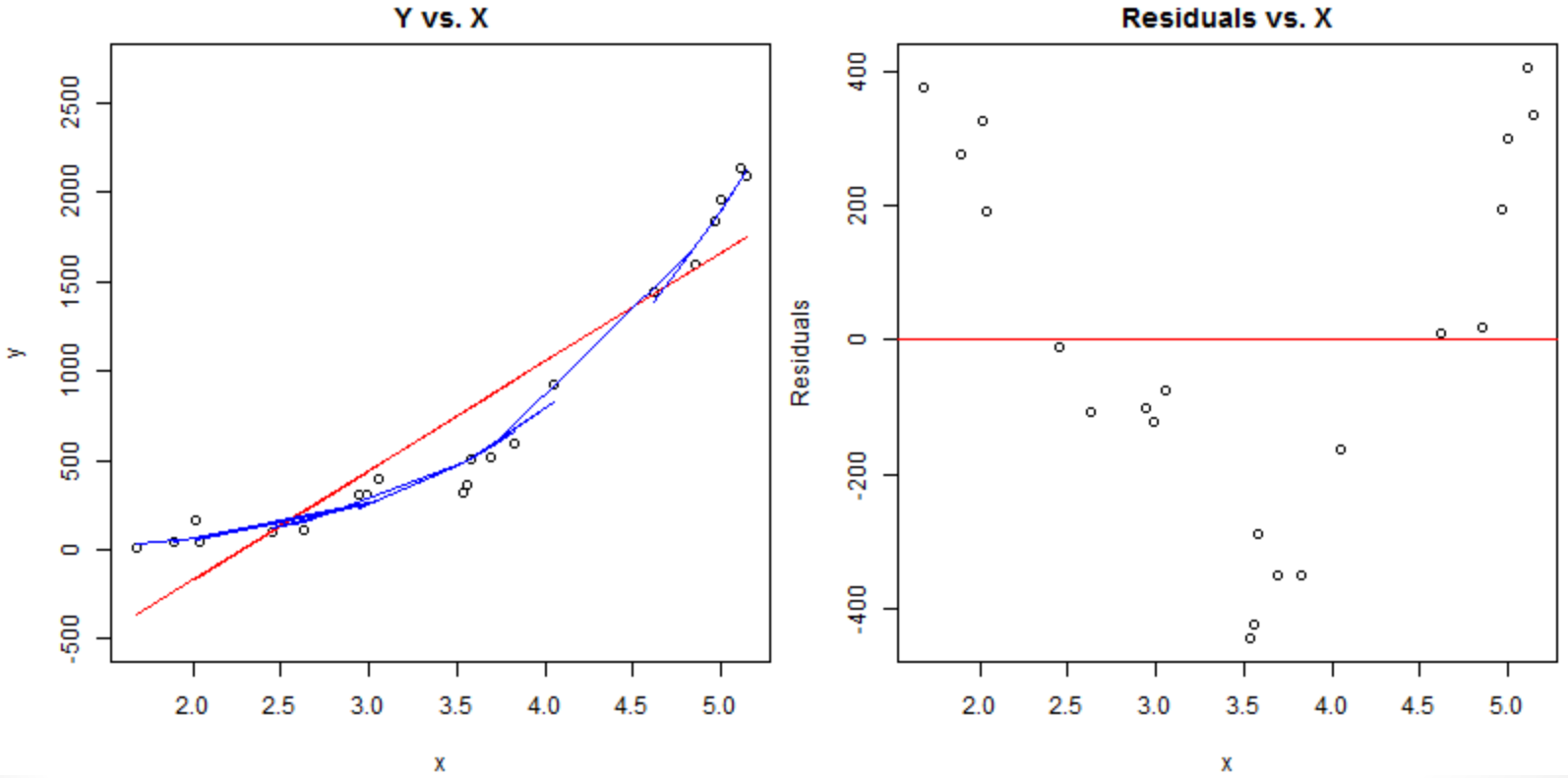
Example – Linearity

- Linearity assumption is satisfied:



Example – Linearity

- Linearity assumption is NOT satisfied:

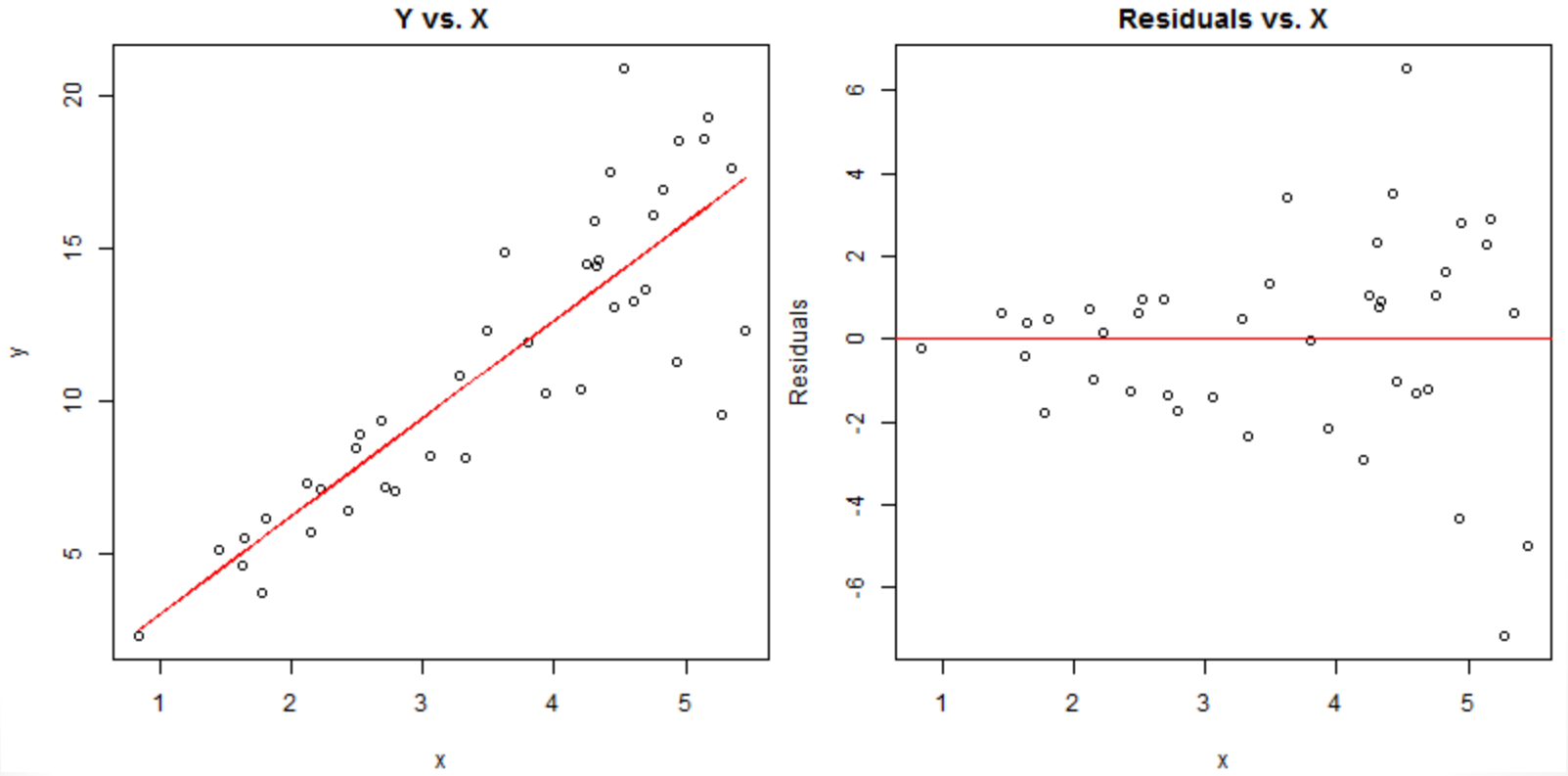


Assumption #2: IID Normal

- The errors $\epsilon_1, \epsilon_2, \ldots, \epsilon_n$ are assumed to be independently and identically distributed (iid) normal with mean 0 and variance σ^2.
  - Check whether the errors have a constant variance.
  - Check whether the errors are normally distributed.
  - Check whether the errors are independent.

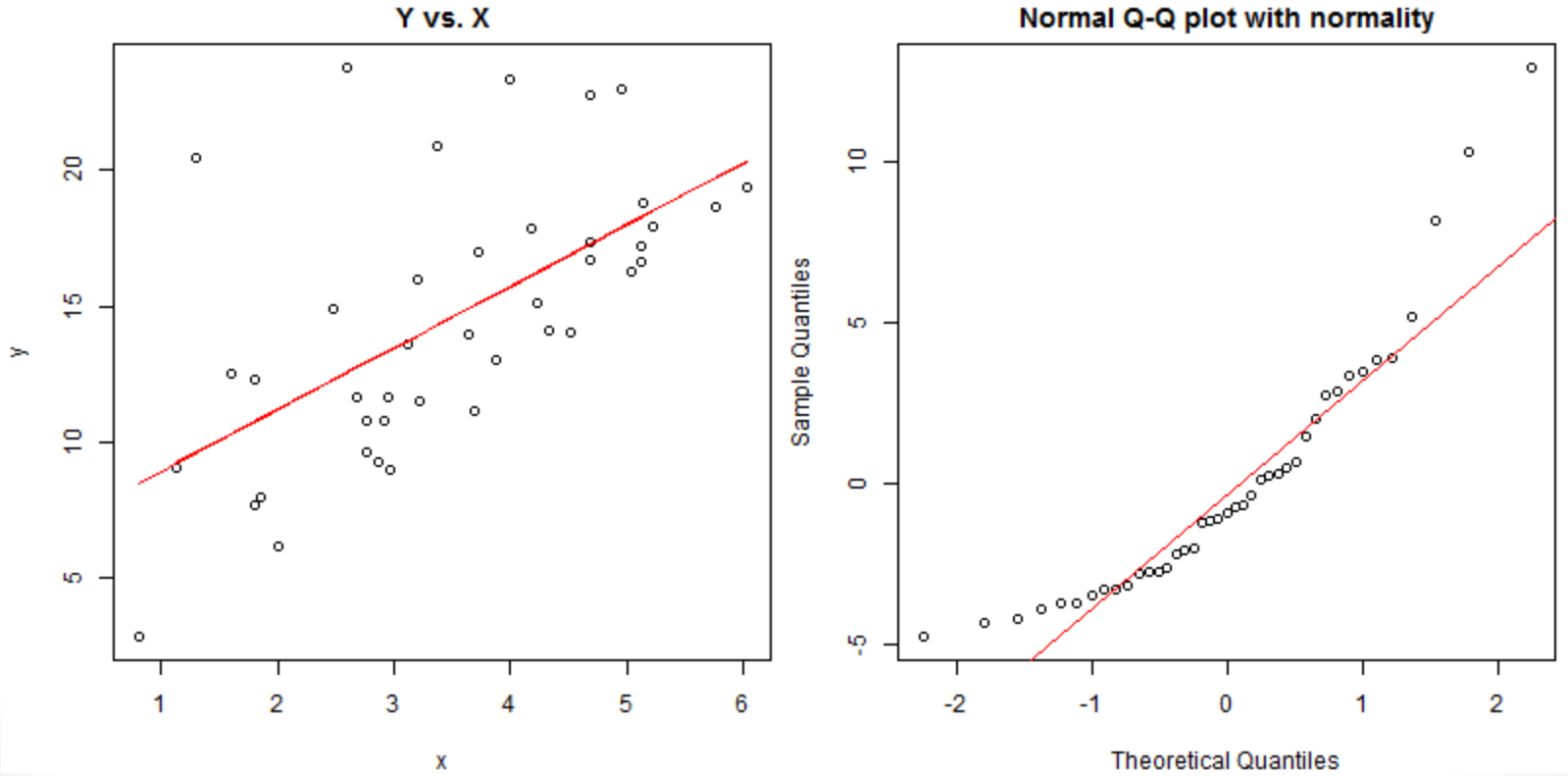
Example – Constant Variance

- The constant variance assumption is violated:



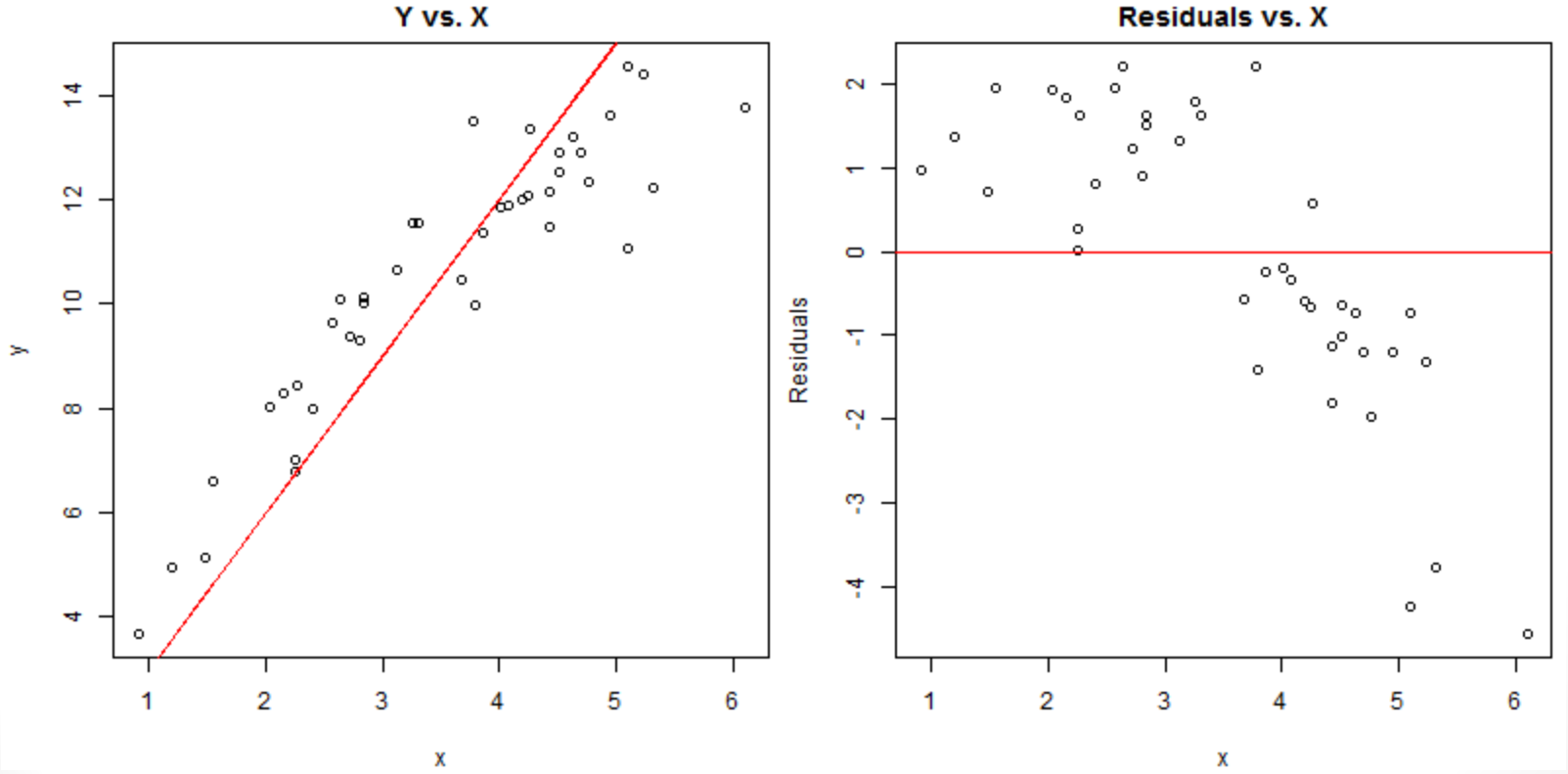
Example - Normality

- The normality assumption is violated:



Example – Independence

- The errors are not independent:

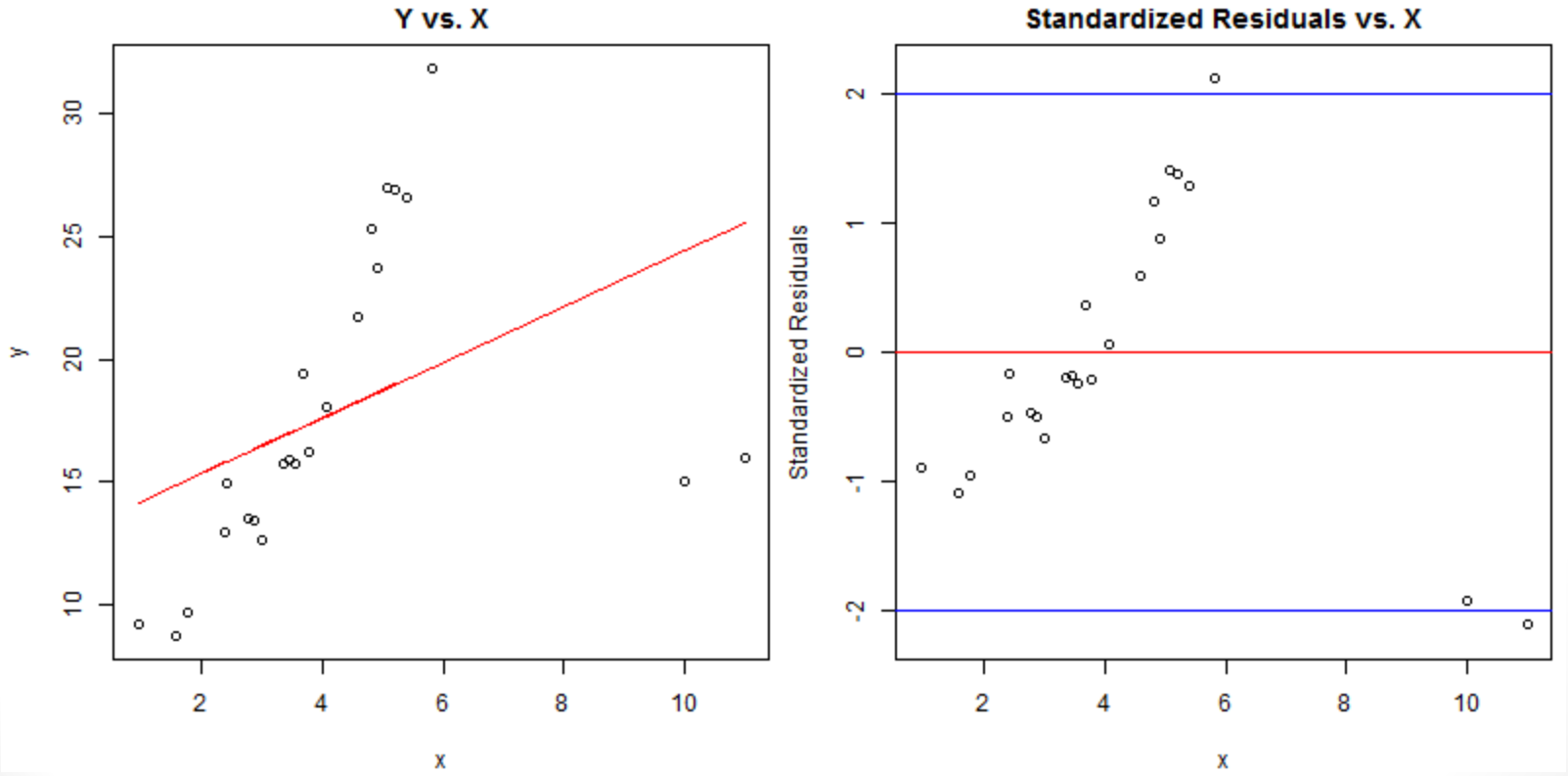


Outliers

- Outliers may significantly affect the regression results.
- To detect outliers, one may use standardized residuals, which are residuals divided by estimates of the standard error of the residuals.
- The observation with the standardized residual greater than 2 or less than -2 is a potential outlier.

Example – Outliers

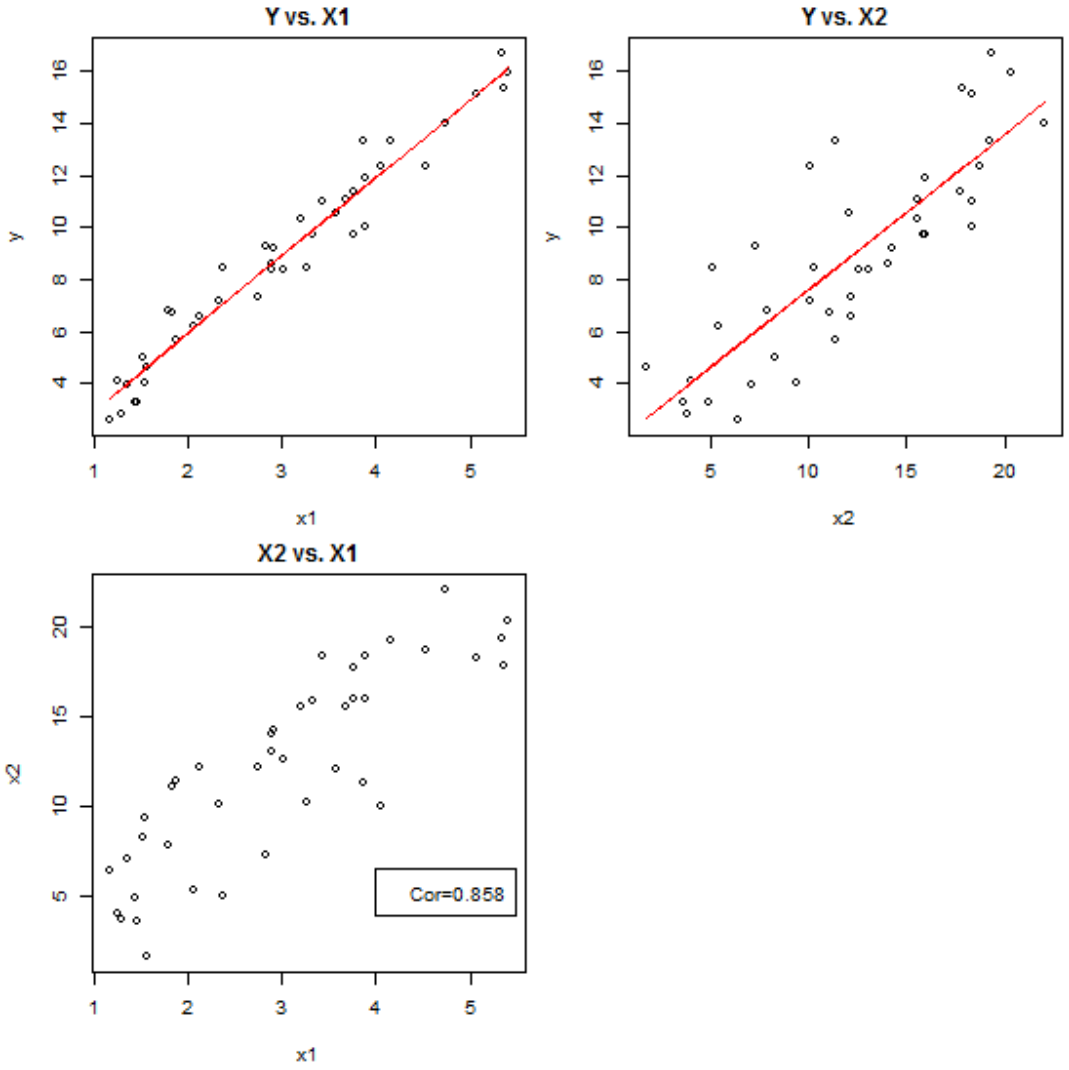
- X=10 and x=11 are outliers.



Multicollinearity

- When the predictor variables are strongly correlated, the regression results may be misleading.
- The estimated coefficients are very sensitive to the addition or deletion of correlated predictors in general.
- The regression coefficients may show large sampling errors, which results in dropping them in the model.
- In practice, multicollinearity may be present if
  - The effect of the predictors is not consistent with what you expected;
  - Predictors that were expected to be significant do now show any significance.

Example - Multicollinearity



Example – Continued

Coefficient of X1 from Y vs. X1	P-value
2.99	<0.001

Coefficient of X2 from Y vs. X2	P-value
0.594	<0.001

Coefficients from Y vs. X1 and X2	P-value
3.036 (X1)	<0.001
-0.012 (X2)	0.804

Remedies

- Data transformation is widely used to cure the violations of linearity, heterogeneity of variance, and normality assumptions. Data transformation needs to be carefully examined.
- For outliers, deletion of them or data transformation is often helpful.

Remedies- Continued

- For correlated errors, time series models are often used to model error terms.
- For collinear data, dropping one of the variables might be helpful. Principal component regression might be useful as well. Increasing sample size is always preferred.

Conclusion

- Graphical methods to check assumptions are reviewed.
- Using linear regression analysis without checking assumptions might draw incorrect results.
- If some of the assumptions are violated, remedies need to be carefully examined.

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  Contact: Jessica Pruszynski, PhD to make an appointment
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Questions?