Basic Concepts of Bayesian Statistics

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Educational Objectives

- Gain an introductory level understanding of what is Bayesian statistics

- Learn essential concepts of Bayesian statistics such as conditional probability; prior, posterior and predictive distributions; credible intervals

- Identify situations in medical research where Bayesian statistics can be particularly useful

Financial Disclosure

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Name:	Role in Meeting:
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Thank you.

Outline

- Interpretation and calibration of probability
- Conditional probability and Bayes Theorem
- Prior, posterior and predictive distributions
- Credible (or posterior probability) intervals
- Testing hypotheses
- When particularly to use Bayesian methods

Probability – Interpretation and Calibration

- Traditional view: inherent property of process; long-run relative frequency
  - A drug's success in a large population
  - Outcomes of games of chance for a casino
- Bayesian view: degree of uncertainty; inherently subjective, depending on available information
  - Chance of snow tomorrow
  - Prediction of outcome for an individual patient

In both viewpoints, conditional probability is an essential concept

Conditional Probability

- Twenty percent of adults smoke: $P(S) = 0.20$
- Forty five percent of smokers are women: $P(W|S) = 0.45$
- Eighteen percent of women smoke: $P(S|W) = 0.18$
- An overwhelming majority of race car drivers are men, yet only a small fraction of men are race car drivers.
- It is crucial to keep target and conditioning events clear and straight

Conditional Probability

The conditioning event focuses attention on a subset of possibilities

$P(S|W) = \frac{P(S \text{ and } W)}{P(W)}$, $P(W) > 0$

$P(S \text{ and } W) = P(W)P(S|W) = P(S)P(W|S)$

Women smokers make up $(0.20)(0.45) = 0.09$ of the population of adults

$0.09 = P(W)(0.18)$ gives $P(W) = 0.09/0.18 = 0.5$

Then $P(M) = 0.5$, $P(M \text{ and } S) = 0.11$ and
$P(S|M) = 0.11/0.5 = 0.22$,
$P(M|S) = 1 - 0.45 = 0.55$

Reversing Target and Conditioning Events: Bayes Theorem

Suppose we know the probability of a symptom S under each of a set $M_1, M_2, \ldots, M_k$ of mutually exclusive and exhaustive medical conditions.

- These medical conditions are deemed to have probabilities $P(M_1), \ldots, P(M_k)$ adding to 1.
- A patient shows symptom S. Given this information, how do we update the probabilities of the medical conditions?

$$\boldsymbol{P(M_i|S) \propto P(M_i)P(S|M_i), \quad i = 1, \ldots, k}$$

$$\boldsymbol{Posterior \ \propto \ Prior \ \times \ Likelihood}$$

Bayes Theorem for Statistical Inference: a Simple Illustration

A snippet from Dr. Jeff Whittle's hypertension study of members of veteran's organizations such as the VFW and the American Legion:

A random sample of 404 vets showed that 184 had uncontrolled hypertension

How does Bayes Theorem help here?

The unkown quantity of interest is the uncontrolled HTN percentage in the entire population of veterans. Call this θ (expressed as a fraction).

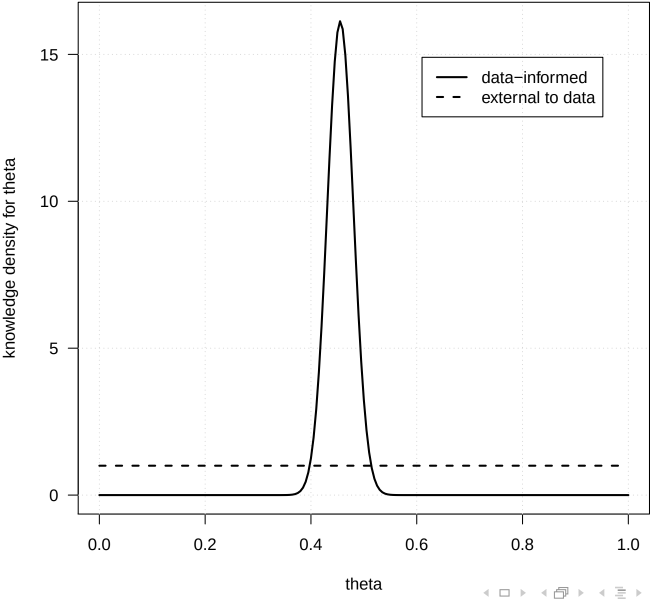
Representing Knowledge: Prior and Posterior Distributions

Knowledge about any unknown is described by a probability distribution

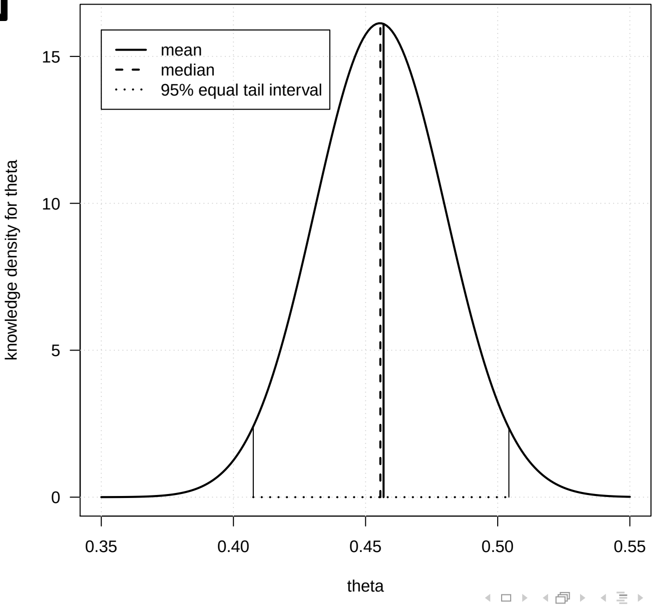
State of knowledge before data collection is called the Prior (data-excluded knowledge distribution)

State of knowledge conditional on data is called the Posterior (data-informed knowledge distribution)

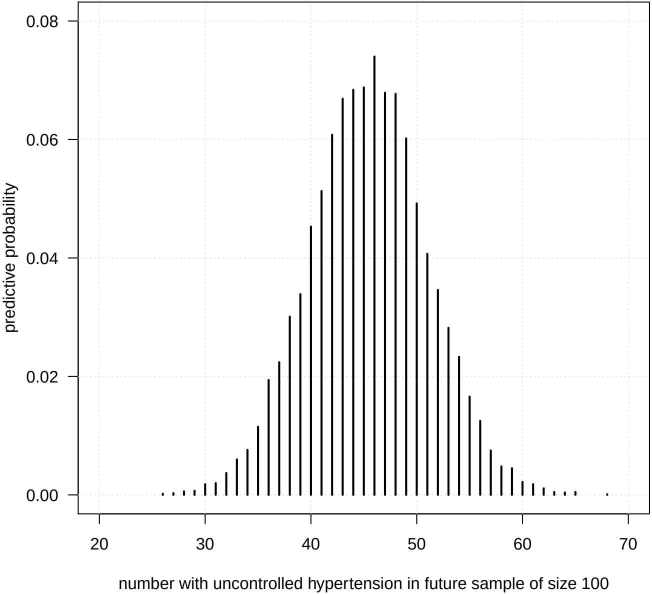
Prior and Posterior: Vet HTN



Posterior (Credible) Interval: Vet HTN



Predictive Distribution: Vet HTN



Comments on Prediction

- Predictive distribution can address a single individual or a group. For example, a physician's patient panel.

- It includes the data-informed uncertainty in the knowledge of the population parameter as well as the patient-to-patient or group-to-group variation.

- Traditionally, prediction has been under-utilized in medical studies. This may be partially due to conceptual issues with traditional methods.

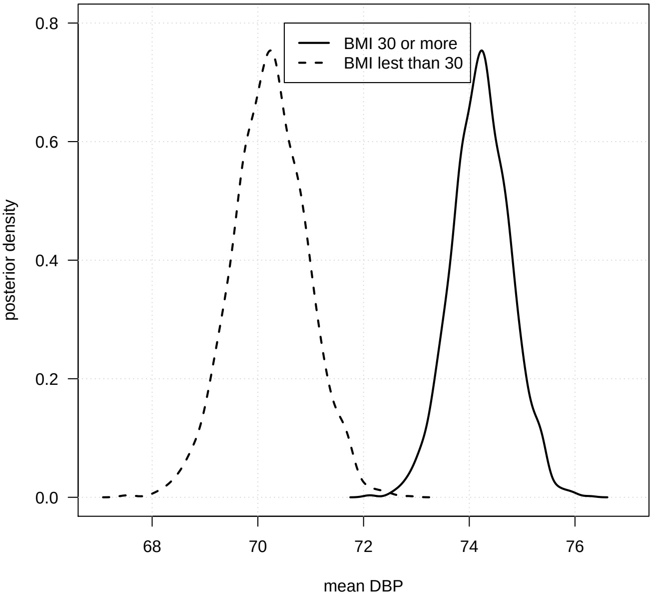
- Prediction is natural and straightforward from the Bayesian viewpoint.

More Freedom in Asking Questions

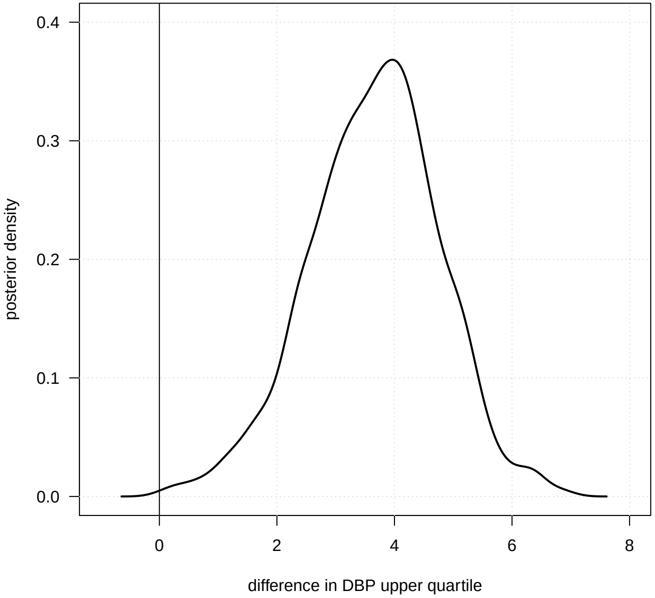
Once the data-informed posterior distribution for the unknown parameter(s) is obtained, it is generally easy to describe the posterior distributions of any other quantities that derive from the parameter(s).

For example, data on heights of two age-groups of children can result in posteriors for $\mu_1, \sigma_1, \mu_2, \sigma_2$. From this we can get posteriors for $\mu_1 - \mu_2$, σ_1/σ_2, $\sigma_1 - \sigma_2$ or even the difference in the two coefficients of variation $\frac{\sigma_1}{\mu_1} - \frac{\sigma_2}{\mu_2}$.

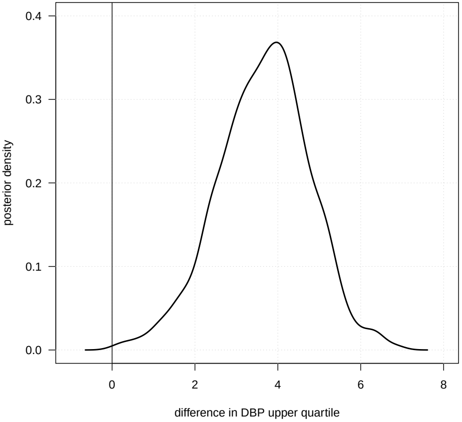
Posterior for Means: Vet HTN

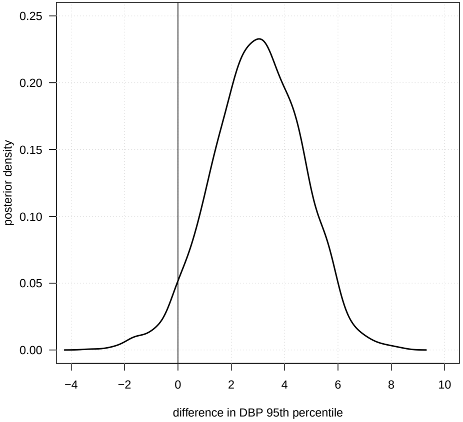


Difference in Quantiles: Vet HTN



Difference in Quantiles: Vet HTN





Accumulating Knowledge

A few years from now, suppose Dr. Whittle conducts another study of the vets.

- Should he begin again with the flat prior, acting as if no knowledge about θ gained from Study I applies to Study II?
- Should he use the posterior from Study I as the prior in Study II?
- Study II prior somewhere between the above two?

Bayesian statistics enables such use of gained knowledge and aims to add to it with new data.

Choice of Prior

- Context dependent; available knowledge about setting needs careful quantification.
- Elicitation of more easily interpretable quantities related to the statistical parameter.
- Use of historical data.
- Expert opinion.
- Information pooling.
- Low-information priors; “no knowledge”; reference priors; objective priors.

Testing Hypotheses

Bayes Theorem provides a straightforward recipe for testing hypotheses such as

$$H_0 : \quad \theta \ < 0.5 \ \text{ versus } \ H_1 : \quad \theta \ \geq 0.5$$

We need these ingredients:

- Prior probability $P(H_0)$ $(P(H_1) = 1 - P(H_1))$
- Prior distributions for θ under each hypotheses
- Observed data

We can then calculate posterior probability $P(H_0|data)$

Testing Hypotheses: Vet HTN

For illustration, take prior for θ to be $U(0, 0.5)$ under H_0 and $U(0.5, 1)$ under H_1.

Data: 184 with uncontrolled HTN among 404 vets

With $P(H_0) = 0.5$,
$P(H_0|data) = 0.96$ and $P(H_1|data) = 0.04$

With $P(H_0) = 0.25$,
$P(H_0|data) = 0.89$ and $P(H_1|data) = 0.11$

Simple, direct interpretation; but depends on $P(H_0)$

Testing Hypotheses: Bayes Factor

We can remove the influence of $P(H_0)$ on the answer by using odds in place of probabilities

Prior odds for H_0 are $P(H_0)/P(H_1)$

Posterior odds for H_0 are $P(H_0|data)/P(H_1|data)$

Ratio of Posterior to Prior odds is free of $P(H_0)$

This is called Bayes Factor,
BF_{01}, in favor of H_0 against H_1

BF in favor of H_1 is $BF_{10} \ = \ 1/BF_{01}$

Note the symmetry of the two hypotheses

Bayes Factor Interpretation

For Vet HTN data, BF in favor of $\theta < 0.5 \; = \; 26.2$

Multiply your prior odds by BF
to get your posterior odds

BF is viewed as weight of evidence in data

Jeffreys's guidelines

BF_{01}	Evidence for H_0 against H_1
1 to 3.2	Not worth more than a bare mention
3.2 to 10	Substantial
10 to 100	Strong
> 100	Decisive

When to use Bayesian Methods: Some Practical Advice

One or more of the following:

- Some focus on prediction
- Important information external to data at hand
- Combining information sources
- Modeling of complex data
- Adaptive clinical trials

Combining Information Sources

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