

# Chapter 1

## Linear Mixed Model

### Implementation

The LMM implementation is available as C source code. It depends on the standard C libraries CHOLMOD (Davis, 2008) and GSL (Galassi et al., 2009). It can be found at <http://www.mcw.edu/biostatistics/Research/Software>

#### 1.1 Data Preparation

A program has been provided for typical data preparation: `onetime.c`. It depends on two header files. The first `gsl_cholmod.h` defines two C preprocessor (cpp) macros that allow GSL to read and write to CHOLMOD matrices present in memory. `MAT(A, B)` is used for matrices and `VEC(A, B)` is used for vectors. They both take two parameters.

A: A pointer to an already existing matrix created by CHOLMOD.

B: The name of a GSL object to create. No new memory will be allocated since A already has memory allocated to it. Therefore, you will not need to “free” the memory associated with B; that will be taken care of when you “free” A instead. The new B object will need

Table 1.1: The definitions of the values found in `onetime.h` .

| Variable        | Definition                                    |
|-----------------|---|
| <code>p</code>  | the dimension of the fixed parameters $\beta$ |
| <code>q1</code> | the number of primary clusters (hospitals)    |
| <code>q2</code> | the number of secondary clusters (surgeons)   |
| <code>q</code>  | the number of total clusters                  |
| <code>N</code>  | the number of subjects (patients)             |

to be operated on as a memory location. For example, if the second parameter is `X`, then you will need to access `&X` .

It is important to note that `CHOLMOD` has a column-major definition of dense matrices: adjacent memory locations represent two adjacent cells in a column. `GSL` has a row-major definition of dense matrices: adjacent memory locations represent two adjacent cells in a row. These differing definitions are easily handled; a column-major matrix is the same as the transpose of a row-major matrix.

The second header file is `onetime.h` . You need to create this header file based on your data. For example, for the XS Scenario, the file looks like Figure 1.1.

```
const int p=4, q1=5, q2=25, q=q1+q2, N=2500;
```

Figure 1.1: An example of `onetime.h` based on the XS Scenario.

See Table 1.1 for the definition of the variable names in `onetime.h`.

The program will read and write several input files in the Matrix Market format (Boisvert et al., 1996); see Table 1.2.

After compiling `onetime.c` into `onetime.out` (or whatever you are calling your executable), then running `onetime.out` will produce the output files and it will print  $\mathbf{y}'\mathbf{y}$  on standard output (`stdout`).

Table 1.2: List of input/output files for `onetime.c` and their definitions.

| Filename:               | content   | definition                              |
|-------------------------|---|---|
| <code>Y.mtx</code>      | the input file for the outcome vector   | $\mathbf{y}$                            |
| <code>uX.mtx</code>     | the input file for the “uncentered” covariates  | $\mathbf{X}$                            |
| <code>X.mtx</code>      | the output file for the “centered” covariates   | $\mathbf{X}$                            |
| <code>XtX.mtx</code>    | the output file for $\mathbf{X}'\mathbf{X}$ where $\mathbf{X}$ has been “centered”  |   |
| <code>XtY.mtx</code>    | the output file for $\mathbf{X}'\mathbf{y}$ where $\mathbf{X}$ has been “centered”  |   |
| <code>uZ.mtx</code>     | the input file for the “unordered”  | $\mathbf{Z}$                            |
| <code>ZtZ.mtx</code>    | the output file for   | $\tilde{\mathbf{Z}}'\tilde{\mathbf{Z}}$ |
| <code>ZtX.mtx</code>    | the output file for $\tilde{\mathbf{Z}}'\mathbf{X}$ where $\mathbf{X}$ has been “centered”  |   |
| <code>ZtY.mtx</code>    | the output file for   | $\tilde{\mathbf{Z}}'\mathbf{y}$         |
| <code>P.mtx</code>      | the output file for the permutation   | $\mathbf{P}$                            |
| <code>u1mask.mtx</code> | An output file for a vector of 1s and 0s. The 1s represent the locations of re-ordered hospitals in $\tilde{\mathbf{u}}$ ; the 0s, re-ordered surgeons. |   |
| <code>D1.mtx</code>     | A similar definition to <code>u1mask.mtx</code> . It is the hospital portion of $\tilde{\mathbf{D}}$ with the surgeon portion zeroed out.               |   |
| <code>D2.mtx</code>     | A similar definition to <code>D1.mtx</code> . It is the surgeon portion of $\tilde{\mathbf{D}}$ with the hospital portion zeroed out.                   |   |

## 1.2 Conjugate Priors

The version of the program created for conjugate priors is `Normal.c`. It depends on two header files. The first `gsl_cholmod.h` has already been discussed. The second header file is `Normal.h`. You need to create this header file based on your data, your prior parameters and your initial values. For example, for the XS Scenario, the file looks like Figure 1.2.

```
const int p=4, q1=5, q2=25, q=q1+q2, N=2500, M=20000;
const double a1=(q1+0.1)/2., b1=0.1,
             a2=(q2+0.1)/2., b2=0.1,
             ae=(N+0.1)/2., be=0.1,
             am=0.,          bm=0.001,
             yty=43137.609891;
double mu=0., taue=1., tau1=1., tau2=1.;
gsl_vector *beta=gsl_vector_calloc(p); // initialized to zeros
```

Figure 1.2: An example of `Normal.h` based on the XS Scenario.

See Table 1.3 for the definition of the variable names in `Normal.h`.

The program will read several input files in the Matrix Market format (Boisvert et al., 1996); see Table 1.4.

After compiling `Normal.c` into `Normal.out` (or whatever you are calling your executable), then running `Normal.out` produces an R source file `Normal.R` containing the Gibbs samples for  $\beta$ ,  $\mu$ ,  $\tau_1$ ,  $\tau_2$  and  $\tau_\epsilon$  (in that order). `Normal.R` is the default name for the output unless you pass a file name as an argument such as `Normal.out example.R`

Table 1.3: The definitions of the values found in `Normal.h`.

| Variable          | Definition  |
|-------------------|---|
| <code>p</code>    | the dimension of the fixed parameters $\beta$                             |
| <code>q1</code>   | the number of primary clusters (hospitals)                                |
| <code>q2</code>   | the number of secondary clusters (surgeons)                               |
| <code>q</code>    | the number of total clusters  |
| <code>N</code>    | the number of subjects (patients)   |
| <code>M</code>    | the number of Gibbs samples to perform                                    |
| <code>a1</code>   | the 1st posterior parameter to the Gamma distribution for $\tau_1$        |
| <code>b1</code>   | the 2nd prior parameter to the Gamma distribution for $\tau_1$            |
| <code>a2</code>   | the 1st posterior parameter to the Gamma distribution for $\tau_2$        |
| <code>b2</code>   | the 2nd prior parameter to the Gamma distribution for $\tau_2$            |
| <code>ae</code>   | the 1st posterior parameter to the Gamma distribution for $\tau_\epsilon$ |
| <code>be</code>   | the 2nd prior parameter to the Gamma distribution for $\tau_\epsilon$     |
| <code>am</code>   | the prior mean for $\mu$  |
| <code>bm</code>   | the prior precision for $\mu$   |
| <code>yty</code>  | $\mathbf{y}'\mathbf{y}$   |
| <code>mu</code>   | the initial value of $\mu$  |
| <code>taue</code> | the initial value of $\tau_\epsilon$                                      |
| <code>tau1</code> | the initial value of $\tau_1$   |
| <code>tau2</code> | the initial value of $\tau_2$   |
| <code>beta</code> | the initial value of $\beta$  |

Table 1.4: List of input files for `Normal.c` and their definitions.

| Filename:               | content definition  |
|-------------------------|---|
| <code>C.mtx</code>      | the prior precision of $\beta$  |
| <code>Cc.mtx</code>     | the prior mean of $\beta$   |
| <code>XtX.mtx</code>    | $\mathbf{X}'\mathbf{X}$ where $\mathbf{X}$ has been “centered”  |
| <code>XtY.mtx</code>    | $\mathbf{X}'\mathbf{Y}$ where $\mathbf{X}$ has been “centered”  |
| <code>ZtZ.mtx</code>    | $\tilde{\mathbf{Z}}'\tilde{\mathbf{Z}}$   |
| <code>ZtX.mtx</code>    | $\tilde{\mathbf{Z}}'\mathbf{X}$ where $\mathbf{X}$ has been “centered”  |
| <code>ZtY.mtx</code>    | $\tilde{\mathbf{Z}}'\mathbf{Y}$   |
| <code>u1mask.mtx</code> | A vector of 1s and 0s. The 1s represent the locations of re-ordered hospitals in $\tilde{\mathbf{u}}$ ; the 0s, re-ordered surgeons.      |
| <code>D1.mtx</code>     | A similar definition to <code>u1mask.mtx</code> . It is the hospital portion of $\tilde{\mathbf{D}}$ with the surgeon portion zeroed out. |
| <code>D2.mtx</code>     | A similar definition to <code>D1.mtx</code> . It is the surgeon portion of $\tilde{\mathbf{D}}$ with the hospital portion zeroed out.     |

### 1.3 Noninformative Prior

The Noninformative prior version of the program, `Normal-Uniform.c`, is very similar to the conjugate prior version. The header file is now named `Normal-Uniform.h`. The definition of the parameters is the same except for those associated with  $\tau_1$  and  $\tau_2$ : `a1`, `b1`, `a2` and `b2`. For example, for the XS Scenario, the file looks like Figure 1.3.

```
const int p=4, q1=5, q2=25, q=q1+q2, N=2500, M=20000;
const double a1=(q1-1)/2., b1=0.01,
              a2=(q2-1)/2., b2=0.01,
              ae=(N+0.1)/2., be=0.1,
              am=0., bm=0.001,
              yty=43137.609891;
double mu=0., taue=1., tau1=1., tau2=1.;
gsl_vector *beta=gsl_vector_calloc(p); // initialized to zeros
```

Figure 1.3: An example of `Normal-Uniform.h` based on the XS Scenario.

After compiling `Normal-Uniform.c` into `Normal-Uniform.out` (or whatever you are calling your executable), then running `Normal-Uniform.out` produces an R source file `Normal-Uniform.R` containing the Gibbs samples for  $\beta$ ,  $\mu$ ,  $\tau_1$ ,  $\tau_2$  and  $\tau_\epsilon$  (in that order). `Normal-Uniform.R` is the default name for the output unless you pass a file name as an argument such as `Normal-Uniform.out example.R`

## Chapter 2

# Logistic Mixed Model

## Implementation

The Logistic Mixed Model implementation is available as C source code. It depends on the standard C libraries CHOLMOD (Davis, 2008) and GSL (Galassi et al., 2009). It can be found at <http://www.mcw.edu/biostatistics/Research/Software>

Although, there are fewer one-time calculations for the Logistic Mixed Model, you can still use the data preparation program previously discussed: `onetime.c`. The only difference is that you need to copy the file `V.mtx` to the non-existent file `Y.mtx` and the program will perform the necessary operations.

### 2.1 Conjugate Priors

The Logistic Mixed Model program, `Logistic.c`, is very similar to the LMM version. The header file is `Logistic.h`. The header file is nearly identical to `Normal.h`; the exceptions are that the parameters `ae`, `be`, `yty` and `taue` are unnecessary. For example, for the XS Scenario the file looks like Figure 2.1.

The program will read several input files in the Matrix Market format (Boisvert et al., 1996); see Table 2.1.

```

const int p=4, q1=5, q2=25, q=q1+q2, N=2500, M=20000;
const double a1=(q1+0.1)/2., b1=0.1,
             a2=(q2+0.1)/2., b2=0.1,
             am=0.,          bm=0.001;
double mu=0., tau1=1., tau2=1.;
gsl_vector *beta=gsl_vector_calloc(p); // initialized to zeros

```

Figure 2.1: An example of `Logistic.h` based on the XS Scenario.

Table 2.1: List of input files for `Logistic.c` and their definitions.

| Filename:               | content   | definition |
|-------------------------|---|------------|
| <code>C.mtx</code>      | the prior precision of $\beta$  |            |
| <code>Cc.mtx</code>     | the prior mean of $\beta$   |            |
| <code>X.mtx</code>      | $\mathbf{X}$ which has been “centered”  |            |
| <code>V.mtx</code>      | $\mathbf{V}$ which is the dichotomous outcome   |            |
| <code>Z.mtx</code>      | $\tilde{\mathbf{Z}}$  |            |
| <code>ZtZ.mtx</code>    | $\tilde{\mathbf{Z}}'\tilde{\mathbf{Z}}$   |            |
| <code>u1mask.mtx</code> | A vector of 1s and 0s. The 1s represent the locations of re-ordered hospitals in $\tilde{\mathbf{u}}$ ; the 0s, re-ordered surgeons.      |            |
| <code>D1.mtx</code>     | A similar definition to <code>u1mask.mtx</code> . It is the hospital portion of $\tilde{\mathbf{D}}$ with the surgeon portion zeroed out. |            |
| <code>D2.mtx</code>     | A similar definition to <code>D1.mtx</code> . It is the surgeon portion of $\tilde{\mathbf{D}}$ with the hospital portion zeroed out.     |            |

After compiling `Logistic.c` into `Logistic.out` (or whatever you are calling your executable), then running `Logistic.out` produces an R source file `Logistic.R` containing the Gibbs samples for  $\beta$ ,  $\mu$ ,  $\tau_1$  and  $\tau_2$  (in that order). `Logistic.R` is the default name for the output unless you pass a file name as an argument such as `Logistic.out example.R`

## 2.2 Noninformative Prior

The Noninformative prior version of the program, `Logistic-Uniform.c`, is very similar to the conjugate prior version. The header file is now named `Logistic-Uniform.h`. The definition of the parameters is the same except for those associated with  $\tau_1$  and  $\tau_2$ : `a1`, `b1`, `a2` and `b2`. For example, for the XS Scenario, the file looks like Figure 2.2.

```
const int p=4, q1=5, q2=25, q=q1+q2, N=2500, M=20000;
const double a1=(q1-1.)/2., b1=0.25,
              a2=(q2-1.)/2., b2=0.25,
              am=0.,          bm=0.1;
double mu=0., tau1=1., tau2=1.;
gsl_vector *beta=gsl_vector_calloc(p); // initialized to zeros
```

Figure 2.2: An example of `Logistic-Uniform.h` based on the XS Scenario.

After compiling `Logistic-Uniform.c` into `Logistic-Uniform.out` (or whatever you are calling your executable), then running `Logistic-Uniform.out` produces an R source file `Logistic-Uniform.R` containing the Gibbs samples for  $\beta$ ,  $\mu$ ,  $\tau_1$  and  $\tau_2$  (in that order). `Logistic-Uniform.R` is the default name for the output unless you pass a file name as an argument such as `Logistic-Uniform.out example.R`

# Bibliography

Boisvert, R., R. Pozo, and K. Remington (1996). The Matrix Market exchange formats: initial design. Technical Report NISTIR 5935, National Institute of Standards and Technology. [<http://math.nist.gov/MatrixMarket>].

Davis, T. (2008). *User guide for CHOLMOD: a sparse Cholesky factorization and modification package*. Department of Computer and Information Science and Engineering, University of Florida, Gainesville, FL. [<http://www.cise.ufl.edu/research/sparse/cholmod>].

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