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The Science of Infinity

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"The infinite! No other question has ever moved so profoundly the spirit of man," said David Hilbert (1862-1943), one of the most influential mathematicians of the 19th century. The subject has been studied extensively by mathematicians and philosophers but still remains an enigma of the intellectual world. The notions of "finite" and "infinite" are primitive, and it is very likely that the reader has never examined these notions very carefully. For example, there are as many even numbers as there are natural numbers. This is true but counter intuitive. We may argue, that the set of even numbers is half the size of the set of natural numbers, thus how can both sets be of the same size?

To answer this question we must first explain what do we mean when we say two sets are the same size? Normally, we would count the elements in each set then verify the counts are the same. However, there is another way to verify two sets are the same size by *matching*. To illustrate, that matching is more fundamental than counting let us consider an example. If I walked into a large room with many chairs but all the seats are occupied and no one is standing then I can conclude that there are as many people as there are chairs. That is, there is a one-to-one correspondence between person and chair. Hence both sets are the same size, even though we did not count how many persons or chairs they were. Similarly, we can match the even numbers to the set of natural numbers through a one-to-one correspondence though endless. Therefore, both sets must be the same size.

In this article we will consider the following questions. What is infinity? Are there different types of infinity? Can it be found in the physical world?

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What is infinity? Infinity is a concept. The idea that something can go on endlessly. It is not a number. To understand infinity we must examine the work of the mathematician Georg Cantor (1845-1918). His ideas were so counter intuitive that he attracted the wrath of some of his contemporaries; prominent mathematicians such as Henri Poincaré (1823-1912) and Leopold Kronecker (1823-1891). Henri Poincaré referred to his ideas as a "grave disease" infecting the discipline of mathematics [1], and Leopold Kronecker described him as a "scientific charlatan" and a "corrupter of youth." [2, 3]

Some infinities are "bigger" than others. Cantor introduced the idea that there are different types of infinity; countable or uncountable. For instance, the set of natural numbers, integers and rationals are countably infinite, meaning we can list all the elements of each set, be it endless. On the other hand, there are infinities that are uncountably infinite such as the real numbers. That is, the elements can not be listed. Cantor proved that there are more numbers between zero and one on the real number line than there are in the entire set of natural numbers. Hence the infinity of the real numbers is "bigger" than the infinities of the natural numbers, integers and rationals. Cantor also showed that if we took all the subsets of the natural numbers we can create an even "bigger" infinity than the original set. We can also take all the subsets of the subsets and create yet another infinity "bigger" than the previous infinity. In other words, there are an infinite 'number' of infinities of different sizes.

We may wonder if infinity exists in the real world. Can infinity be found in the physical world? Surprisingly, the idea of infinity can be found in the Bible. Psalm 90:2 reads, "from eternity to eternity, you are God."–*New World Translation* Hence like the real numbers, God has no beginning and no end. He has always existed. Another example can be found in neuroscience. In regard to memory, for instance, our brain has an enormous capacity. In *The Brain Book*, Peter Russell writes: "Memory is not like a container that gradually fills up, it is more like a tree growing hooks onto which the memories are hung. Everything you remember is another set of hooks on which more new memories can be attached. So the capacity of memory keeps on growing. The more you know, the more you can know."

Cantor wondered, is there an infinity between the infinity of the natural numbers and the infinity of the real numbers? This conjecture is known as the continuum hypothesis. In the 1920's Kurt Godel (1906-1978) showed that you can never prove that the continuum hypothesis is false. However, in the 1960's Paul J. Cohen (1934-2007) showed that you can never prove that the continuum hypothesis is

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true. Therefore this question remains unanswered. However the apparent contradiction could mean we do not have all the fundamental principles yet to answer this question. We may be missing some key principles yet to be discovered.

References

[1] Dauben, Joseph W. (1979), Georg Cantor: his mathematics and philosophy of the infinite, Boston: Harvard University Press, page 266.

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[3] Dauben, Joseph W. (1977), "Georg Cantor and Pope Leo XIII: Mathematics, Theology, and the Infinite", Journal of the History of Ideas 38 (1): 85-108.

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